Mathematics Stage 5 (Year 10) – unit of learning

Applying exponentials

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# Rationale

The NSW Department of Education publishes a range of curriculum support materials, including samples of lesson sequences, scope and sequences, assessment tasks, examinations, student and teacher resource booklets, and curriculum planning and curriculum evaluation templates. The samples are not exhaustive and do not represent the only way to complete or engage in each of these processes. Curriculum design and implementation is a dynamic and contextually-specific process. While the mandatory components of syllabus implementation must be met by all schools, it is important that the approach taken by teachers is reflective of their needs, and faculty or school processes.

The NSW Education Standards Authority (NESA) defines [programming](https://educationstandards.nsw.edu.au/wps/portal/nesa/k-10/understanding-the-curriculum/programming) as ‘the process of selecting and sequencing learning experiences which enable students to engage with syllabus outcomes and develop subject specific skills and knowledge’ (NESA 2022). A program is developed collaboratively within a faculty. It differs from a unit in important ways, as outlined by NESA on their [Advice on units](https://educationstandards.nsw.edu.au/wps/portal/nesa/k-10/understanding-the-curriculum/programming/advice-on-units) page. A unit is a contextually-specific plan for the intended teaching and learning for a particular class for a particular period. The organisation of the content in a unit is flexible and it may vary according to the school, the teacher, the class and the learning space. They should be working documents that reflect the thoughtful planning and reflection that takes place during the teaching and learning cycle. There are mandatory components of programming and unit development, and this template provides one option for the delivery of these requirements. The NESA and department guidelines that have influenced this template are elaborated upon at the end of the document.

This resource has been developed to assist teachers in NSW Department of Education schools to create learning that is contextualised to their classroom. It can be used as a basis for the teacher’s own program, assessment, or scope and sequence, or be used as an example of how the new curriculum could be implemented. The resource has suggested timeframes that may need to be adjusted by the teacher to meet the needs of their students.

# Overview

**Description**: this program of learning addresses content from the focus areas of Financial Mathematics B and Non-linear relationships A, B and C. The lessons and sequences in this program of learning are designed to allow students to explore exponential functions in a variety of contexts, including compound interest and depreciation.

**Duration**: this program of learning is designed to be completed over a period of approximately 5 weeks but can be adapted to suit the school context.

**Explicit teaching**: suggested learning intentions and success criteria are available for some lessons provided. Learning intentions and success criteria are most effective when they are contextualised to meet the needs of students in the class. The examples provided in this document are generalised to demonstrate how learning intentions and success criteria could be created.

**Accessing the resources**: this program of learning includes a range of student-facing and teacher resources. All resources can be accessed from the [Stage 5 Unit 11 – applying exponentials](https://education.nsw.gov.au/teaching-and-learning/curriculum/mathematics/mathematics-curriculum-resources-k-12/mathematics-7-10-resources/stage-5-unit-11-applying-exponentials) catalogue page.

# Outcomes

## Core

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* solves financial problems involving compound interest and depreciation **MA5-FIN-C-02**
* identifies connections between algebraic and graphical representations of quadratic and exponential relationships in various contexts
**MA5-NLI-C-01**
* **identifies and compares features of parabolas and exponential curves in various contexts MA5-NLI-C-02**

## Path

A student:

* interprets and compares non-linear relationships and their transformations, both algebraically and graphically **MA5-NLI-P-01**

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**Prior to planning for teaching and learning, please consider the following**:

**Engagement**

* How will I provide authentic, relevant learning opportunities for students to personally connect with lesson content?
* How will I support every student to grow in independence, confidence, and self-regulation?
* How will I facilitate every student to have high expectations for themselves?
* How will I identify and provide the support each student needs to sustain their learning efforts?

**Representation**

* What are some different ways I can present content to enable every student to access and understand it?
* How will I identify and address language and/or cultural considerations that may limit access to content for students?
* How will I make lesson content and learning materials more accessible?
* How will I plan learning experiences that are relevant and challenging for the full range of students in the classroom?

**Expression**

* How will I provide multiple ways for students to respond and express what they know?
* What tools and resources can students use to demonstrate their understanding?
* How will I know every student has understood the concepts and language presented in each lesson?
* How will I monitor if every student has achieved the learning outcomes and learning growth?

# Lesson sequence and details

## Learning episode 1 – simple and compound interest

### Teaching and learning activity

Students define compound interest as repeated applications of simple interest by exploring an investment of candies.

### Syllabus content

* Examine compound interest for up to 3 time periods using repetition of the formula for simple interest
* Associate the calculation of the total value of a compound interest investment with repeated multiplication, using digital tools
* Establish and use the formula $FV=PV\left(1+r\right)^{n}$ to find compound interest where FV = future value of the investment, PV= present value of the investment, $r=$ interest rate per time period and $n=$ number of time periods
* Solve problems involving compound interest
* Compare simple interest with compound interest in practical situations
* Recognise non-linear relationships in real-life contexts and solve related problems

Table 1 – lesson sequence and details

|  |  |  |
| --- | --- | --- |
| Teaching and learning activities | Required resources | Registration, adjustments and evaluation notes |
| Simple and compound interest Duration**:** 2 lessonsLearning intentions* To understand the difference between simple and compound interest.
* To be able to calculate compound interest.

Success criteria* I can calculate repeated applications of simple interest.
* I can calculate compound interest using the formula $FV=PV\left(1+r\right)^{n}$.
* I can solve problems involving the formula $FV=PV\left(1+r\right)^{n}$.
* I can explain the difference between simple and compound interest.
 | * *Simple and compound interest* PowerPoint
* Appendix A and C, printed (one copy per pair)
* Appendix B, printed (one copy per student)
* Optional: candy
* Optional: container
 |  |

## Learning episode 2 – bad investments

### Teaching and learning activity

Students explore the depreciation formula $S=V\_{0}\left(1-r\right)^{n}$ to investigate whether cars are a good investment.

### Syllabus content

* Use the compound interest formula to establish the depreciation formula $S=V\_{0}\left(1-r\right)^{n}$ where $S=$ salvage value, $V\_{0}=$ initial value of the asset, $r=$ depreciation rate per time period and $n=$ number of periods
* Solve problems involving the depreciation of an asset

Table 2 – lesson sequence and details

|  |  |  |
| --- | --- | --- |
| Teaching and learning activities | Required resources | Registration, adjustments and evaluation notes |
| Bad investments Duration**:** 1 to 2 lessonsLearning intentions* To be able to calculate the depreciated value of an asset.
* To understand the declining balance method of depreciation.

Success criteria* I can explain what is meant by the term ‘depreciation’.
* I can calculate the depreciated value of an asset using the declining balance depreciation formula.
* I can explain the connection between compound interest and declining balance depreciation.
* I can justify why some asset’s values decrease rapidly when new.
 | * Digital device (per pair of students)
* *Bad investments* PowerPoint
* *Bad investments* Excel
* Appendices A and B, printed A3 (one per group of 3)
 |  |

## Learning episode 3 – Euler’s number

### Teaching and learning activity

Students explore the effect that increasing the frequency of compounding periods has on the overall interest. They then explore how the frequency of compounding interest relates to Euler’s number $e$.

### Syllabus content

* Establish and use the formula $FV=PV(1+r)\^n$ to find compound interest where FV = future value of the investment, PV= present value of the investment, $r=$ interest rate per time period and $n=$ number of time periods
* Solve problems involving compound interest
* Use the compound interest formula to establish the depreciation formula $S=V\_{0}\left(1-r\right)^{n}$ where $S=$ salvage value, $V\_{0}=$ initial value of the asset, $r=$ depreciation rate per time period and $n=$ number of periods
* Solve problems involving the depreciation of an asset

Table 3 – lesson sequence and details

|  |  |  |
| --- | --- | --- |
| Teaching and learning activities | Required resources | Registration, adjustments and evaluation notes |
| **Euler’s number** **Duration:** 1 to 2 lessons**Learning intention*** To understand how increasing the compounding periods affects interest and depreciation.

**Success criteria*** I can calculate compound interest when compounded at different periods.
* I can describe how interest and depreciation change as compound frequency changes.
* I can explain how compound interest relates to Euler’s number $e$.
 | * *Euler’s number* PowerPoint
* Appendix A, printed (one per student)
* Appendix B, printed and cut into cards, (one per pair, without digital device option)
* Optional: digital device (per pair of students)
 |  |

## Learning episode 4 – invasive species

### Teaching and learning activity

Students explore the growth rate of invasive species to Australia, creating exponential equations and exploring exponential graphs.

### Syllabus content

* Construct a table of values to graph non-linear relationships involving quadratics and exponentials
* Identify graphs and equations of parabolas and exponential curves
* Graph and compare exponential curves of the form $y=a^{x} $using graphing applications
* Recognise quadratics and exponentials in real-life contexts
* Describe features of exponential curves including the $y$-intercept, asymptote and the nature of the curve for very large and very small values of $x$
* Associate graphs of straight lines, parabolas and exponential curves with the appropriate equations
* Identify and describe features of different types of graphs based on their equations (Path)

Table 4 – lesson sequence and details

|  |  |  |
| --- | --- | --- |
| Teaching and learning activities | Required resources | Registration, adjustments and evaluation notes |
| Invasive species Duration**:** 1 to 2 lessonsLearning intention* To understand the relationship between exponential equations and their graphs.

Success criteria* I can determine the equation for an exponential pattern.
* I can explain how the equation of an exponential affects the graph.
 | * *Invasive species* PowerPoint
* Optional: digital device to complete Appendix D (per group of 3)
* Appendix A, B and C, printed (one copy per pair)
* Appendix D, printed (one copy per group of 3)
 |  |

## Learning episode 5 – exponential Marbleslides

### Teaching and learning activity

Students make use of Desmos’ Marbleslide activities to explore transformations with exponential equations.

### Syllabus content

* Represent quadratic and exponential relationships using graphing applications
* Graph and compare exponential curves of the form $y=a\^x $using graphing applications
* Describe features of exponential curves including the $y$-intercept, asymptote and the nature of the curve for very large and very small values of $x$
* Use graphing applications to graph exponential relationships of the form $y=k\left(a\right)^{x}+c$ and $y=k\left(a\right)^{-x}+c$ for integer values of $k$, $a$ and $c$ (where $a>0$ and $a\ne 1$), and compare and describe any relevant features (Path)

Table 5 – lesson sequence and details

|  |  |  |
| --- | --- | --- |
| Teaching and learning activities | Required resources | Registration, adjustments and evaluation notes |
| **Exponential Marbleslides** **Duration:** 1 lesson**Learning intention*** To be able to graph exponential relationships in the form $y=k(a)^{x}+c$.

**Success criteria*** I can identify the y-intercept of an exponential graph.
* I can write the equation for an exponential graph’s horizontal asymptote.
* I can describe how graphs in the form $y=k(a)^{x}+c$ differ from $y=a^{x}.$
 | * One device (per pair)
* *Exponential Marbleslides* PowerPoint
 |  |

## Learning episode 6 – free falling

### Teaching and learning activity

Students explore exponential relationships through an experiment and question what occurs as $x$ becomes extremely small or large.

### Syllabus content

* Describe features of exponential curves including the $y$-intercept, asymptote and the nature of the curve for very large and very small values of $x$
* Recognise non-linear relationships in real-life contexts and solve related problems
* Identify and describe features of different types of graphs based on their equations (Path)

Table 6 – lesson sequence and details

|  |  |  |
| --- | --- | --- |
| Teaching and learning activities | Required resources | Registration, adjustments and evaluation notes |
| **Free falling** **Duration:** 1 to 2 lessons**Learning intention*** To understand what happens to an exponential relationship as $x$ gets really large.

**Success criteria*** I can use an exponential relationship to model a real-world scenario.
* I can explain what happens to the $y$-values as the $x$-values increase in an exponential relationship.
 | * Digital device (per student)
* Measuring tape (per group of 3)
* Coloured table tennis ball or handball (per group of 3)
* Stopwatch
 |  |

## Learning episode 7 – population clock

### Teaching and learning activity

Students use the population clock to explore the limitations of exponential relationships when used to model everyday occurrences. Students then interpret information from different models and identify any limitations.

### Syllabus content

* Recognise quadratics and exponentials in real-life contexts
* Describe features of exponential curves including the $y$-intercept, asymptote and the nature of the curve for very large and very small values of $x$
* Associate graphs of straight lines, parabolas and exponential curves with the appropriate equations
* Recognise non-linear relationships in real-life contexts and solve related problems

Table 7 – lesson sequence and details

|  |  |  |
| --- | --- | --- |
| Teaching and learning activities | Required resources | Registration, adjustments and evaluation notes |
| Population clock Duration**:** 1 lessonLearning intention* To understand when exponential graphs are a good model to represent real-life scenarios.

Success criteria* I can use an exponential model to predict future values.
* I can identify limitations of models that use exponential relationships.
* I can interpret information from an exponential graph.
 | * *Population clock* PowerPoint
* Appendix A and B, printed (one per student)
 |  |

# References

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