# Describing geometrical patterns

Students explore geometric patterns using matchsticks. The focus is on being able to informally describe the relationship between the number of the term and the number of matchsticks.

## Visible learning

### Learning intention

* To be able to describe a geometric pattern using words.

### Success criteria

* I can identify a geometric pattern.
* I can describe a geometric pattern in words.
* I can represent geometric patterns using a table of values.
* I can find further terms in a geometric pattern.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* creates and displays number patterns and finds graphical solutions to problems involving linear relationships **MA4-LIN-C-01**

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Table 1: lesson summary

|  |  |  |  |
| --- | --- | --- | --- |
| Section | Summary of activity | Teaching strategies | Teaching points |
| Warm up | Students complete ‘Route to Infinity’ from NRICH ([bit.ly/RouteToInfinity](https://bit.ly/RouteToInfinity)) using a ‘0 to 25 Coordinate Grid’ ([bit.ly/Grid\_Paper\_mathlinks](https://bit.ly/Grid_Paper_mathlinks)). | Visibly random groups of 3  Vertical non-permanent surfaces  Gallery walk | Use Working mathematically skills to activate prior learning of plotting points and identifying patterns. |
| Launch | Students notice and wonder about the matchstick pattern displayed on slide 3 of the PowerPoint *Describing geometrical patterns* (DGP PPT). Students add the meaning of the word ‘term’ to their glossary. | Notice and wonder  Pose-Pause-Pounce-Bounce | Learn to identify and describe patterns. |
| Explore | Students investigate an increasing geometric pattern using matchsticks and complete [Appendix A](#_Appendix_A_1). Students are challenged to find the number of matchsticks for the 100th term in the pattern. | Think-Pair-Share  Assessing and advancing questions  Turn and talk | Identify the need to move from concrete to visual representations to a general rule in words. Move from additive thinking in patterns to multiplicative thinking. |
| Summarise | Students investigate different strategies for rearranging matchstick patterns displayed on slides 5–10 of the PowerPoint (DGP PPT) and use these strategies to develop a general rule for the pattern. | Think-Pair-Share  Pose-Pause-Pounce-Bounce | Explore different strategies to assist in writing a general rule in words for geometric patterns. |
| Apply | The teacher displays instructions from slide 12 of the PowerPoint (DGP PPT) for students to complete [Appendix B](#_Appendix_B) in groups. The teacher then displays slides 13–14 of the PowerPoint for students to find the rules for the reversed, decreasing patterns. | Visibly random groups of 3  Vertical non-permanent surfaces  Pose-Pause-Pounce-Bounce  Gallery walk | Complete tables of values and describe geometric patterns in words Then make comparisons with the same pattern when it is reversed. |

## Activity structure

Please use the associated PowerPoint *Describing geometrical patterns* (DGP PPT) to display images in this lesson.

### Warm up

1. Display the poster ‘Route to Infinity’ from NRICH ([bit.ly/RouteToInfinity](https://bit.ly/RouteToInfinity)) and read through the challenge together.

The poster image on the page will need to be selected to enlarge it for students to view.

1. Assign students to visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) at vertical non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)).
2. Distribute to each group of 3, an A3 sheet of ‘0 to 25 Coordinate Grid’ paper ([bit.ly/Grid\_Paper\_mathlinks](https://bit.ly/Grid_Paper_mathlinks)) in a plastic sleeve with adhesive putty.
3. Ask students to investigate and write down responses to the 3 questions on the poster, as well as prepare an explanation of their strategy.
4. Students undertake a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)) of the responses. Ask one member of each group to stay with their responses to explain the group’s strategy.

At this point, walk around with the students and take note of strategies and responses to be highlighted later in a class discussion.

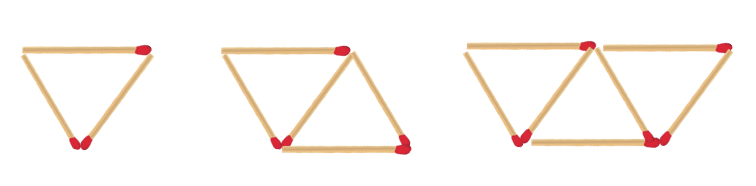
1. Conduct a class discussion highlighting different strategies and showing solutions.

Use the ‘Student Solutions’ tab on the ‘Route to Infinity’ NRICH page ([bit.ly/RouteToInfinity](https://bit.ly/RouteToInfinity)), to view solutions and different strategies for solving the problem.

### Launch

1. With students sitting back at their desks, display slide 3 from the PowerPoint (DGP PPT).

Figure 1: matchstick pattern on slide 3



Adapted from Transum (1997–2024)

1. Use a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)) for students to discuss what they notice and wonder ([bit.ly/noticewonderstrategy](https://bit.ly/noticewonderstrategy)) about the matchstick pattern.

* Students might notice that the pattern is made up of triangles. They might count the number of matchsticks in each term of the pattern, and they might see that one side of the triangle is shared with the next triangle.
* They might wonder about the pattern continuing, what the next one looks like and how many matchsticks it takes to make it.

1. Initiate a sharing of ideas and reasoning using the Pose-Pause-Pounce-Bounce questioning strategy (PDF 557 KB) ([bit.ly/posepausepouncebounce](https://bit.ly/posepausepouncebounce)). Ask students to describe how this pattern is growing.

Students might describe the pattern as adding on 2 more matchsticks each time.

1. Explain to students that we use the word ‘term’ to describe the location of a number or image in a pattern or sequence.
2. Ask students to add the word ‘term’ to the glossary page in their books along with its definition. Students started a glossary page in Lesson 1 – plotting points.

### Explore

This activity was adapted from NZ Maths unit ‘Matchstick Patterns’ ([bit.ly/MatchstickPatterns](https://bit.ly/MatchstickPatterns)).

1. Display slide 3 from the PowerPoint (DGP PPT) again and ask students ‘How can we find the number of matchsticks for the 100th term in the pattern? What about the 500th term in the pattern?’
2. In a Think-Pair-Share, provide students with 30 matchsticks and Appendix A ‘Matchstick strategies’. Ask students to find the number of matchsticks for each term in the table.

The aim of this activity is to move students from additive thinking to multiplicative thinking. By only providing students with 30 matchsticks, they will not have enough to be able to continue to make the terms in the problem but should be able to identify what stays the same in each term and what changes.

1. Ask students assessing and advancing questions ([bit.ly/supportingstrategies](https://bit.ly/supportingstrategies)) to further student thinking. Some suggestions are provided in the following table.

Table 2: assessing and advancing questions

|  |  |
| --- | --- |
| Assessing questions | Advancing questions |
| What do you know about this pattern of matchsticks? | How can you find other terms in the pattern? |
| Can you describe the pattern? | What patterns have you noticed in the table? |
| What strategies have you tried so far? | What stays the same each time in the pattern? What is changing? |
| How did you know which numbers to write in the bottom row of the table? | Do you think there is any link between the numbers in the columns? |

Students might focus on the number pattern (3, 5, 7) in the bottom row of the table and try to continue the pattern to find the 100th term. Question students about the efficiency of doing this. Is there a better way?

1. Ask pairs to complete a turn and talk [(bit.ly/classroomtalkmoves](https://bit.ly/classroomtalkmoves)) with another pair to discuss how they found the 100th term. If students haven’t managed to find the 100th term, they can discuss what they have discovered so far.

### Summarise

Slides 7 and 10 of the PowerPoint (DGP PPT) include an algebraic rule; however, these can be removed if students are not ready to move toward an algebraic rule. The focus of the lesson is on describing the pattern in words.

1. Display slide 5 of the PowerPoint (DGP PPT) and explain that this is how John decided to rearrange the matches to make it easier to work out the number of matchsticks in each term.
2. Display slide 6 of the PowerPoint (DGP PPT) and use a Think-Pair-Share for students to discuss the question prompts on the slide.

This activity aims to move from the repeated adding of 2s to multiplying by the number of 2s. Students should notice that the number of 2s is the same as the term number.

1. In pairs, ask students to use this strategy to find the number of matchsticks for the 15th, 20th, and 50th terms of the pattern.
2. Ask pairs to write down a rule for John’s strategy that they could give students in another class to assist them to find out the number of matchsticks for any term.
3. Use the Pose-Pause-Pounce-Bounce questioning strategy for pairs to share their rule and for the class to discuss the advantages and disadvantages of each one.
4. Display slide 7 of the PowerPoint (DGP PPT) which shows a rule written in words. Discuss the rule with students and any similarities to the rules they came up with.
5. Display slide 8 of the PowerPoint (DGP PPT) and explain that Brendan rearranged the matchsticks differently to make counting easier.
6. Display slide 9 of the PowerPoint (DGP PPT) and use a Think-Pair-Share for students to discuss the question prompts on the slide.

Students should see a relationship in which the number of 2s is one less than the term number.

1. In pairs, ask students to use this strategy to find the number of matchsticks for the 15th, 20th and 50th terms of the pattern.
2. Initiate a sharing of ideas and reasoning using the Pose-Pause-Pounce-Bounce questioning strategy. Some suggestion prompts might include:

* Did you get the same answers as you did with John’s strategy?
* What about when you used your own strategy?

1. Again, ask pairs to write down a rule for Brendan’s strategy that they could give to students in another class to assist them to find the number of matchsticks for any term.

This rule is more complicated than the first rule, but students should still attempt to explain it in words.

1. Display slide 10 of the PowerPoint (DGP PPT) which shows a rule written in words and using algebra. Discuss the rule with students and draw similarities to the rules they came up with.

### Apply

1. Place students back into their visibly random groups of 3 on vertical non-permanent surfaces.
2. Distribute Appendix B ‘Matchstick patterns’ to each group. Display and read through the instructions on slide 12 of the PowerPoint (DGP PPT).

While students are working on the activity, find opportunities to bring groups who are using different strategies together to check the reasonableness of their solutions.

Pattern 3 is non-linear. Students aren’t expected to find the 99th term in the pattern. They should focus on being able to describe the pattern and ways they could find the 99th term, such as using a spreadsheet.

In the sample solutions, the algebraic rule has been provided for students who are ready to move towards this concept.

1. Display slide 13 of the PowerPoint (DGP PPT) and ask groups to complete a table of values for the first 3 terms in the pattern and describe the pattern in words.

Figure 2: matchstick pattern on slide 13

A matchstick pattern as follows:
first term: 12 squares made of 32 matchsticks
second term: 10 squares made of 27 matchsticks
third term: 8 squares made of 22 matchsticks.

1. Ask students to compare their answers and strategies for this pattern with pattern 1 in Appendix B ‘Matchstick patterns’.
2. Use the Pose-Pause-Pounce-Bounce questioning strategy for groups to share and discuss what changes occurred when the pattern was reversed.

Students should notice that the pattern is now decreasing by the same amount it was increasing before.

Students may realise that they can change the sign from positive to negative for the decreasing amount in their original algebraic rule, which means they will also need to change the amount that is added at the end of their rule to get the total number of matchsticks.

1. Display slide 14 of the PowerPoint (DGP PPT) and ask groups to complete a table of values for the first 3 terms in the pattern, describe the pattern in words and write an algebraic rule to describe the pattern.

Figure 3: matchstick pattern on slide 14

A matchstick pattern of triangles and squares.
First term: 23 matchsticks with 6 squares in the middle and 2 triangles at the left and right ends of the squares.
Second term: 20 matchsticks with 5 squares in the middle and 2 triangles at the left and right ends of the squares.
Third term: 17 matchsticks with 4 squares in the middle and 2 triangles at the left and right ends of the squares.

1. Students undertake a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)) of the responses to compare their solutions.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Warm up**

* Students could be provided with a coordinate point in the pattern.
* Students could be provided with the ‘0 to 10 blank Coordinate Grid’ grid paper ([bit.ly/Grid\_Paper\_mathlinks](https://bit.ly/Grid_Paper_mathlinks)) and choose their scales.
* Students could create their own path pattern challenge and swap with another group.

**Launch**

* Students could be provided with mini whiteboards ([bit.ly/miniwhiteboards](https://bit.ly/miniwhiteboards)) or matchsticks to help them understand the pattern.

**Explore**

* Less ready students may require more matchsticks to create further iterations of the pattern before moving to a visual representation.
* Students may require further assessing and advancing questions.
* Students can work within the full spectrum for describing patterns, from informally and with the use of concrete representations up to algebraic terms.

**Summarise**

* Less ready students could explore John’s strategy of rearranging the pattern.
* Students could be challenged to find a third way of rearranging the pattern. See the extension idea under Session 1: Triangle paths at ([bit.ly/MatchstickPatterns](https://bit.ly/MatchstickPatterns)).
* Students can work within the full spectrum for describing patterns, from informally and with the use of concrete representations up to algebraic terms.

**Apply**

* Less ready students can be provided with matchsticks as support to complete Appendix B and the reversed patterns.
* Students can complete further matchstick patterns from ([bit.ly/TransumMatchsticks](https://bit.ly/TransumMatchsticks)).
* Students can be challenged to find other non-linear patterns.

### Suggested opportunities for assessment

**Warm up**

* This activity provides an opportunity to assess students’ readiness by revising and applying their learning from Lesson 1 – plotting points.
* A gallery walk allows students to self and peer-assess following the activity.

**Launch**

* Observe students’ reasoning and justification in their responses during the Pose-Pause-Pounce-Bounce questioning.

**Explore**

* Students give each other peer feedback, before sharing with the class in a Think-Pair-Share.
* Appendix A could be collected as evidence of finding the number of matchsticks for higher terms in a pattern.

**Summarise**

* Students will demonstrate their working mathematically skills in discussions, when making comparisons and justifications.

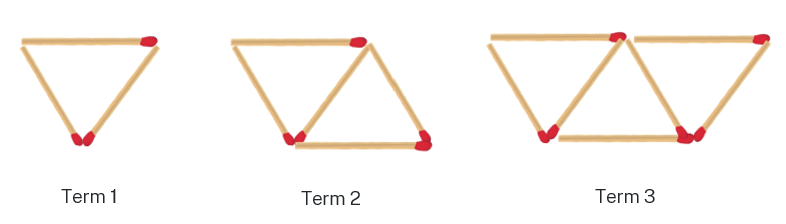
**Apply**

* Appendix B could be collected to observe students’ understanding of describing geometric patterns, completing a table of values and being able to find larger terms in patterns.
* Students working at vertical non-permanent surfaces means the teacher can assess student progress and provide support where appropriate.
* When placed in groups of 3, students provide and receive peer feedback on their understanding.

## Appendix A

### Matchstick strategies

Calculate the number of matchsticks in each term.



Adapted from Transum (1997–2024)

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Term number** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 10 | 15 | 20 | 50 |
| **Number of matchsticks** |  |  |  |  |  |  |  |  |  |  |  |  |

## Appendix B

### Matchstick patterns

Part of this activity has been adapted from Transum’s activity Matchstick Patterns Level 1 from ([bit.ly/TransumMatchsticks](https://bit.ly/TransumMatchsticks)).

#### Pattern 1

A matchstick pattern as follows:
first term: 2 squares made of 7 matchsticks
second term: 4 squares made of 12 matchsticks
third term: 6 squares made of 17 matchsticks.

Adapted from Transum (1997–2024)

**Table of values**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Term number (t)** | 1 | 2 | 3 | 99 |
| **Number of matchsticks (M)** |  |  |  |  |

**Describe the pattern in words:**

#### Pattern 2

**A matchstick pattern of triangles and squares.
First term: 5 matchsticks making 2 triangles pointing away from each other and sharing a base.
Second term: 8 matchsticks with a square in the middle and 2 triangles on the left and right of the square.
Third term: 11 matchsticks with 2 squares in the middle and 2 triangles at the left and right ends of the squares.**

Adapted from Transum (1997–2024)

**Table of values**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Term number (t)** | 1 | 2 | 3 | 99 |
| **Number of matchsticks (M)** |  |  |  |  |

**Describe the pattern in words:**

#### Pattern 3

A matchstick pattern.
First term: 1 horizontal matchstick
Second term: 1 horizontal matchstick with 2 matchsticks pointing off each end. Total of 5 matchsticks.
Third term: 1 horizontal matchstick, 2 matchsticks pointing off each end and then another 2 matchsticks pointing off these ends. Total of 13 matchsticks.

**Table of values**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Term number (t)** | 1 | 2 | 3 | 99 |
| **Number of matchsticks (M)** |  |  |  |  |

**Describe the pattern in words:**

## Sample solutions

### Appendix A – matchstick strategies

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Term number** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 10 | 15 | 20 | 50 |
| **Number of matchsticks** | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 21 | 31 | 41 | 101 |

### Appendix B – matchstick patterns

#### Pattern 1

**Table of values**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Term number (t)** | 1 | 2 | 3 | 99 |
| **Number of Matchsticks (M)** | 7 | 12 | 17 | 497 |

**Word descriptions:**

The pattern is increasing by 5 each time.

Number of matchsticks = 5 times the number of the term + 2 more

**Algebraic rule:**

Students might choose to use John’s strategy by splitting up the shape as follows:

|  |  |  |
| --- | --- | --- |
| First term | Two squares matchstick pattern split up into 2 groups. One with 2 vertical matchsticks and then the other with 5 matchsticks |  |
| Second term | Four squares matchstick pattern split up into 3 groups. One with 2 vertical matchsticks the next one with 5 matchsticks and the last group with 5 matchsticks. |  |
| Third term | Six squares matchstick pattern split up into 4 groups. One with 2 vertical matchsticks the next one with 5 matchsticks, then another group of 5 matchsticks and the last group with 5 matchsticks. |  |

Using this strategy the 99th term would be .

#### Pattern 2

**Table of values**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Term number (t)** | 1 | 2 | 3 | 99 |
| **Number of Matchsticks (M)** | 5 | 8 | 11 | 299 |

**Word descriptions:**

The pattern is increasing by 3 each time.

Number of matchsticks = 5 + (the number of the term − 1) × 3

**Algebraic rule:**

Students might choose to use Brendan’s strategy by splitting up the shape as follows:

|  |  |  |
| --- | --- | --- |
| First term | Two triangle matchstick pattern with 5 matchsticks. |  |
| Second term | Two triangles and one square matchstick pattern split up into 3 groups. First group is a 3 matchstick triangle. Second group is 3 matchsticks (top, bottom and one side of square). Third group is 2 matchsticks (sides of a triangle). |  |
| Third term | Two triangles and 2 squares matchstick pattern split up into 4 groups. First group is a 3 matchstick triangle. Second group is 3 matchsticks (top, bottom and one side of square). Third group is 3 matchsticks (top, bottom and one side of square). Last group is 2 matchsticks (sides of a triangle). |  |

Using this strategy the 99th term would be .

#### Pattern 3

**Table of values**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Term number (t)** | 1 | 2 | 3 | 99 |
| **Number of Matchsticks (M)** | 1 | 5 | 13 | Approximately |

**Word description:** the pattern is increasing by doubling itself, 4, 8, 16, and so on.

|  |  |  |
| --- | --- | --- |
| First term | A picture of a matchstick. | 1 |
| Second term | A picture of a matchstick pattern. There is a single matchstick then there are 4 matchsticks laid out with each one pointing diagonally to a corner. |  |
| Third term | A picture of a matchstick pattern. There is a single matchstick then there are 3 sets of 4 matchsticks laid out with each one pointing diagonally to a corner. |  |

Using this strategy, students should notice that they can’t find a way to express this pattern in a generalised way to find the number of matches for the 99th term.

Students may notice the following pattern:

First term: 1

Second term:

Third term:

**Algebraic rule:**

Using this strategy the 99th term would be .

#### Reversed patterns

Pattern 1 – slide 13 of PowerPoint (DGP PPT)

**Table of values**

|  |  |  |  |
| --- | --- | --- | --- |
| **Term number (t)** | 1 | 2 | 3 |
| **Number of Matchsticks (M)** | 32 | 27 | 22 |

**Word description:**

The pattern is decreasing by 5 each time.

Number of matchsticks = -5 × the number of term + 37

**Algebraic rule:**

Pattern 2 – slide 14 of PowerPoint (DGP PPT)

**Table of values**

|  |  |  |  |
| --- | --- | --- | --- |
| **Term number (t)** | 1 | 2 | 3 |
| **Number of Matchsticks (M)** | 23 | 20 | 17 |

**Word description:**

The pattern is decreasing by 3 each time.

Number of matchsticks = −3 × the number of term + 26

**Algebraic rule:**

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Transum (1997–2024) [*Matchstick Patterns: Level 1*](https://www.transum.org/maths/activity/matchstick_patterns/?Level=1), Transum website, accessed 26 November 2024.

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