# Surplus factors

Students explore finding abundant numbers. They find the factors of terms and use Venn diagrams to find the highest common factor (HCF) of terms.

## Visible learning

### Learning intention

* To be able to find the highest common factor (HCF) within a set of terms.

### Success criteria

* I can find the factors of a term.
* I can find the factors of an algebraic term.
* I can identify common factors of a set of terms.
* I can explain how to find the highest common factor.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* generalises number properties to operate with algebraic expressions including expansion and factorisation **MA4-ALG-C-01**
* operates with primes and roots, positive-integer and zero indices involving numerical bases and establishes the relevant index laws **MA4-IND-C-01**

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Table 1: lesson summary

|  |  |  |  |
| --- | --- | --- | --- |
| ****Section**** | ****Summary of activity**** | ****Teaching strategies**** | ****Teaching points**** |
| ****Warm-up**** | Use slide 3 of the PowerPoint Surplus factorsto understand how to solve a product puzzle and then complete puzzles from [Appendix A](#_Appendix_A). | Think-Pair-Share | Students should recognise that the squares contain factors of the numbers written around the outside of the square. |
| ****Launch**** | Define abundant numbers using the information on slide 5 of the PowerPoint Surplus factors. Then practise finding the factors of numbers by discovering abundant numbers. | Think-Pair-ShareVisibly random groups of 3Vertical non-permanent surfacesGallery walksPose-Pause-Pounce-Bounce | Students are encouraged to consider what strategies may be effective in finding the factors of a number. |
| ****Explore**** | Revisit finding factors of numbers, including algebraic factors. Use Venn diagrams, on slides 7 to 9 of the PowerPoint Surplus factors to find common factors of terms and the HCF. | Pose-Pause-Pounce-BounceVisibly random groups of 3Vertical non-permanent surfacesGallery walksNotice and wonder | Venn diagrams are useful for finding the HCF. |
| ****Summarise**** | Students practise finding the highest common factors through the banner task questions in [Appendix B](#_Appendix_B). | Visibly random groups of 3Vertical non-permanent surfacesBanner tasks | The questions in the banner task mostly involve algebraic terms. Students should find using Venn diagrams useful to identify the HCF. |
| ****Apply**** | Students use the cartoon on slide 11 of the PowerPoint Surplus factors to consider perfect and deficient numbers and create a Venn diagram that includes a deficient number. | Visibly random groups of 3Vertical non-permanent surfacesPose-Pause-Pounce-BounceGallery walks | When creating a Venn diagram, students will need to consider whether to decide on the terms first or to draw the diagram first. |

### Activity structure

Please use the associated PowerPoint Surplus factors to display images in this lesson.

### Warm-up

1. Display slide 3 of the PowerPoint Surplus factors, which shows a product puzzle.
2. Ask students to use a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)) to discuss what they think the numbers in the squares might be and what they represent.
3. Distribute Appendix A ‘Product puzzles’ to pairs of students and ask them to complete the puzzles.

### Launch

1. Display slide 5 from the PowerPoint Surplus factors and ask students, in a Think-Pair-Share, to decide what they think an abundant number is.

An abundant number is a positive integer whose sum of its factors (excluding itself) is greater than the number.

1. Initiate a sharing of ideas and reasoning using the Pose-Pause-Pounce-Bounce questioning strategy (PDF 557 KB) ([bit.ly/posepausepouncebounce](https://bit.ly/posepausepouncebounce)) and use the discussion to ensure students remember what a factor is.

A factor is a number that divides another number without a remainder.

1. Assign students into visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) at a vertical non-permanent surface ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)).
2. Ask students to find the abundant numbers less than 50.
3. Students should then do a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)) to observe the approach of other groups to the question.

The first abundant number is , which has factors of and :

The abundant numbers under 50 are 12, 18, 20, 24, 30, 36, 40, 42 and 48.

1. Initiate a sharing of ideas and reasoning using the Pose-Pause-Pounce-Bounce questioning strategy to discuss how students found the numbers. Questions could include:
* How did you approach the problem?
* What visual representations might be useful to find the factors of numbers?
* Why were you able to eliminate or exclude some numbers?
* What are some of the characteristics of an abundant number?

Students should notice that prime numbers can never be abundant, as their sum will only be one. Students should notice that numbers with many factors are more likely to be abundant.

Students could suggest that factor trees may be a useful visual representation.

### Explore

1. By continuing to work in their same visibly random groups of 3 on vertical non-permanent surfaces, ask students to find the factors of 42.

The factors of are and .

Students may suggest listing them in order. However, a useful strategy may be to pair them up, as multiples, to make .

Teachers could challenge students to consider negative factors as well.

1. Ask students to find the factors of .

Students should recognise that the factors of are: .

1. Students could then do a gallery walk to observe and compare the strategies used to find the factors.
2. Show slide 7 from the PowerPoint Surplus factors. This slide shows a Venn diagram with the factors of in one section and the factors of in the other. The section that overlaps in the centre is for numbers that are factors of both and .
3. Have students transfer their lists of factors to the top of their vertical non-permanent surface and then copy the Venn diagram from the PowerPoint to their surface before adding the numbers to the appropriate sections.
4. Use a Pose-Pause-Pounce-Bounce questioning strategy to discuss the Venn diagram. Questions could include:
* What do we call the factors listed in the centre part of the diagram?
* What is the Highest Common Factor of and ? How do we know?

Students have come across the concept of the highest common factor (HCF) in Lesson 1 – highest common factor of integers of Unit 3 – representing numbers.

1. Display slide 8 of the PowerPoint Surplus factors.
2. Ask students to copy and complete the Venn diagram and use it to find the common factors of and .

**Section A**:

**Section B**: 2, 6, 9, 10, 18, 30, 45, 90

**Section C**: 1, 3, 5, 15

**HCF**: 15

1. Use a Pose-Pause-Pounce-Bounce questioning strategy to facilitate a class discussion using the following question prompts:
* Why do all the terms in Section A contain a pronumeral?
* What are the common factors of and 90?
* What is the highest common factor of and 90?
1. Display slide 9 of the PowerPoint Surplus factors to discuss what they notice and wonder ([bit.ly/noticewonderstrategy](https://bit.ly/noticewonderstrategy)) about the Venn diagram displayed. Useful question prompts may include:
* How could we know what the terms were?
* Why is there an in the common zone and another on the right side?
* How could we determine the HCF?

The Venn diagram shows only the prime factors, rather than listing all the factors. The terms are and and the HCF is .

1. By continuing to work in the same visibly random groups of 3, ask students to find the factors for the terms and and use a Venn Diagram to determine the highest common factor.

**Zone A**: 3,

**Zone B**: 2, 3,

**Zone C**: 2, 2,

**HCF**:

### Summarise

1. Establish new visibly random groups of 3 at vertical non-permanent surfaces.
2. Instruct students to set up their board to complete a banner task ([bit.ly/supportingstrategies](https://bit.ly/supportingstrategies)).
3. Read out the instructions and write up Question 1 from Appendix B ‘Banner task questions’.

For this task, students can use factor trees, Venn diagrams or another suitable method.

### Apply

1. Display slide 11 of the PowerPoint Surplus factors, showing a conversation about perfect numbers and deficient numbers.
2. In a Pose-Pause-Pounce-Bounce, ask students ‘If an abundant number is a number in which the sum of its factors, not including itself, is greater than the number, what do you think a perfect number and a deficient number might be?’
3. With students continuing to work in visibly random groups of 3 at vertical non-permanent surfaces, ask students to find the deficient numbers under 50.
4. In a Pose-Pause-Pounce-Bounce, ask students what they notice about the factors of deficient numbers compared to the factors of abundant numbers.

Students should notice that deficient numbers have very few factors. For example, 35 only has 1, 5 and 7, which adds to 13, which is less than 35, whereas 46 only has 1, 2 and 23, which adds to 26, which is less than 46.

1. Ask students to draw a Venn diagram in which one term is a deficient number, the HCF is 5 and the other term is algebraic.
2. Use a gallery walk for students to see the Venn diagrams created by other groups.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Launch**

* **Enable students to explain what abundant numbers are by providing them with some examples of abundant numbers and their factors.**
* Provide students with a calculator or multiplication grid to help find factors.
* **Ask students if they can find abundant numbers greater than 50.**

**Explore**

* Modify the numbers to have fewer factors to enable students.
* Students can use visual representations, such as factor trees, to assist in finding all the factors of a number. Prime number circles from Amplify’s Polypad (<https://polypad.amplify.com/p>) may assist students.
* Students may need to explicitly review how to complete a factor tree from Lesson 1 – highest common factor of integers of Unit 3 – representing numbers.

**Summarise**

* Teachers might circle examples or working that is exemplary on the vertical non-permanent surfaces for other students to view and take note of.
* **Teachers may decide which questions to allocate from the list of banner task questions.**

**Apply**

* Less-ready students may benefit from being encouraged to complete a factor tree and Venn diagram to find the deficient numbers.

### Suggested opportunities for assessment

**Warm-up**

* Use the completion of the product puzzles to assess student readiness for the lesson.

**Explore**

* When placed in visibly random groups of 3, students provide and receive peer feedback on their understanding.
* Check that students can find all factors of a number.
* Monitor student responses to the Venn diagram questions, checking that students can identify all factors and can identify the HCF.

**Summarise**

* Students will demonstrate their Working mathematically skills in discussions and justifications.

**Apply**

* Teachers can collect Appendix A to use as evidence of their ability to find common factors of numeric and algebraic terms.
* Teachers can record the Venn diagram task as evidence of their understanding of factors and the HCF.

## Appendix A

### Product puzzles

Complete the product puzzles.







## Appendix B

### Banner task questions

Find the highest common factor for each set of terms.

1. and
2. and
3. and
4. and
5. and
6. , and
7. , and
8. , and

## Sample solutions

### Appendix A – product puzzles

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| A 2 by 2 grid with 2 numbers below it and 2 numbers on the right.   The box has 8 and 3 on the bottom, and 6 and 4 on the side.   It has the numbers 2, 3, 4 and 1 inside it.  | A 2 by 2 grid with 2 numbers below it and 2 numbers on the right.   The box has 16 and 27 on the bottom, and 24 and 18 on the side.   It has the numbers 8, 3, 2 and 9 inside it.  | A 2 by 2 grid with 2 numbers below it and 2 numbers on the right.   The box has 12a and 24 on the bottom, and 16a and 18 on the side.   It has the numbers 4a, 4, 3 and 6 inside it.  | A 2 by 2 grid with 2 numbers below it and 2 numbers on the right.   The box has 33a and 56a on the bottom, and 21a and 88a on the side.   It has the numbers 3a, 7, 11 and 8a inside it.  | A 2 by 2 grid with 2 numbers below it and 2 numbers on the right.   The box has 10a squared and 6ab on the bottom, and 6a squared and 10ab on the side.   It has the numbers 2a, 3a, 5a and 2b inside it.  | A 2 by 2 grid with 2 numbers below it and 2 numbers on the right.   The box has 12ab and 25ab on the bottom, and 15a squared and 20b squared on the side.   It has the numbers 31, 5a, 4b and 5b inside it.  |

### Appendix B – banner task questions

1. − HCF =
2. − HCF =
3. − HCF =
4. − HCF = 5
5. − HCF =
6. − HCF =
7. − HCF =
8. − HCF =

## References

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