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Mathematics K–6 support materials for students with special education needs

Case study 2

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# Introduction

The Year 6 class teacher is planning a unit of work for the Number and Algebra Strand (Substrand: Multiplication and Division). In developing a whole class program to meet the learning needs of all students, the teacher needs to consider the particular learning needs of one student, Robert.

Robert is experiencing difficulty acquiring the necessary knowledge, skills and understanding in all strands of the Mathematics Syllabus due to gaps in his background knowledge and skills, and lack of fluency with key facts.

Robert is often reluctant to engage with learning experiences in Mathematics. When he finds a task difficult or is undertaking new learning, he exhibits off-task behaviours and task refusal. Robert experiences difficulty in setting out and organising his work.

## Adjustments to be trialled

In discussion with Robert, the teacher has determined that the following adjustments will be trialled to support Robert’s learning across the key learning areas:

* pre-teaching of background knowledge and skills
* additional time to practise background knowledge and skills (eg 6, 7, 8, and 9 times tables) in a small group or with a peer
* use of scaffolds to assist with his setting out of work
* planned opportunities for Robert to experience a high level of success with new learning
* provision of immediate feedback to Robert, including verbal praise
* tracking, using graphs, of his progress during the skill acquisition and fluency phases of learning to increase his motivation
* positioning of Robert in the classroom to minimise distractions, facilitate monitoring, and to provide the opportunity for him to view appropriate peer models.

The target audience for this content is: Teachers, Principals

* [Evidence of learning](#_Evidence_of_learning)
* [Feedback](#_Feedback)
* [Determining the starting point for instruction](#_Determining_the_starting)
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# Determining the starting point for instruction

The teacher has gained initial information about the students’ knowledge, skills and understanding in the Number and Algebra Strand (Substrand: Multiplication and Division) from their Year 5 reports.

The students were recently assessed to determine their achievement in relation to Stage 2 and Stage 3 outcomes in the Multiplication and Division Substrand, and the starting point for instruction.

The assessment has indicated to the teacher that Robert can:

* describe multiplication as finding the product of two or more numbers
* recall multiplication facts fluently for 1, 2, 3, 4, 5, and 10 times tables
* understand place-value concepts (ones, tens, hundreds, thousands, and trading)
* use mental strategies to multiply a two-digit number by a one-digit number
* use a formal written algorithm, applying place value, to solve addition and subtraction problems involving two-, three- and four-digit numbers (when provided with grid paper).

# Selection of outcomes and content

Using the evidence of learning from the assessment for Multiplication and Division, the following outcomes and content were selected for the class.

| **Outcomes** | Syllabus content |
| --- | --- |
| MA3-2WM Selects and applies appropriate problem-solving strategies, including the use of digital technologies, in undertaking investigationsMA3-6NA Selects and applies appropriate strategies for multiplication and division, and applies the order of operations to calculations involving more than one operation | Solve problems involving [multiplication](http://syllabus.nesa.nsw.edu.au/mathematics/mathematics-k10/glossary/#multiplication) of large numbers by one- or two-digit numbers using efficient mental and written strategies and appropriate digital technologies (ACMNA100)* use mental and written strategies to multiply two- and three-digit numbers by two-digit numbers, including:
	+ using an area model for two-digit by two-digit multiplication, eg 25 × 26Area model for two-digit by two-digit multiplication
	+ [factorising](http://syllabus.nesa.nsw.edu.au/mathematics/mathematics-k10/glossary/#factor) the numbers, eg 12 × 25 = 3 × 4 × 25 = 3 × 100 = 300
	+ using the extended form (long multiplication) of the formal algorithm, egExtended form (long multiplication) of the formal algorithm
* apply appropriate mental and written strategies, and digital technologies, to solve multiplication word problems
	+ use the appropriate [operation](http://syllabus.nesa.nsw.edu.au/mathematics/mathematics-k10/glossary/#operation) when solving problems in real-life situations (Problem Solving)
* record the strategy used to solve multiplication word problems
	+ use selected words to describe each step of the solution process (Communicating, Problem Solving)

Use estimation and rounding to check the reasonableness of answers to calculations (ACMNA099)* use estimation to check the reasonableness of answers to multiplication and division calculations, eg ‘32 × 253 will be about, but more than, 30 × 250’
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# Teaching strategies for the learning experiences and assessment opportunities

The teacher:

* verbalises each step while modelling the procedure
* has students talk through each step of the procedure during guided practice
* initially provides high levels of monitoring and assistance to ensure that students complete the steps of the procedure correctly
* provides a permanent model, with the correct setting out, of a completed procedure on a chart or on the whiteboard
* scaffolds examples so that students initially work only on problems involving multiplication facts that they can recall fluently. This enables students to focus on the steps of the procedure
* could break the procedure into steps, with students practising a particular step until that step has been mastered. The teacher then introduces the next step, requiring students to complete the steps that they have mastered, together with the new step, in the correct sequence
* makes multiplication grids available for reference if students are unable to recall a fact quickly
* provides students with a variety of different types of multiplication problems, eg one-digit by two-digit, two-digit by two-digit, and two-digit by three-digit, when students can accurately complete the procedure.

# Learning experiences and assessment opportunities

As part of this unit, the teacher is planning to implement the following learning experiences and assessment opportunities. The teacher has documented the adjustments that Robert needs in order to access the planned learning experiences and assessment opportunities.

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| Extended Form of MultiplicationStudents multiply numbers by breaking the calculation into two partseg 32 × 21 = 32 × 20 + 32 × 1Students are shown how these parts can be combined when using the extended form (without trading) of multiplication for two-digit numbers by two-digit numbers.*Part A*The teacher models the procedure as follows:*Step 1:* The teacher poses the problem: ‘Jason swims 32 laps per day for 21 days. How many laps does he swim altogether?’*Step 2:* The teacher poses the question: ‘What operation should I use to solve the problem?’*Step 3:* The teacher says, ‘To solve this problem I can use the extended form of multiplication (“long multiplication”)’.*Step 4:* The teacher says, ‘Before solving the problem using the extended form of multiplication, I estimate the answer. I know that 30 multiplied by 20 is 600, so I know that my answer will be more than 600.’*Step 5:* The teacher writes the following on the board:32 x 21 The teacher says, ‘I write the larger number first and the smaller number underneath, making sure that the ones in the bottom number are under the ones in the top number, and that the tens in the bottom number are under the tens in the top number. I write the multiplication sign to the left of the bottom number and provide space for working out and the answer.’*Step 6:* The teacher models this step of the procedure by pointing to numbers as they are used, and recording the results. The teacher says, ‘The first line in our working is where I record the result of multiplying the top number by the number of ones in the bottom number. I multiply 32 by 1. To do this I first multiply 2 by 1, and record 2 in the ones column. Then I multiply 3 (which represents 3 tens or 30) by 1 and record 3 in the tens column.’*Step 7:* The teacher models this step of the procedure by pointing to numbers as they are used, and recording the results. The teacher says, ‘The second line in our working is where I record the result of multiplying the top number by the number of tens in the bottom number. I multiply 32 by 2 tens (or 20). To do this, I first multiply 2 in the top number by 2 (tens) and record 4 in the tens column. Zero is recorded in the empty ones column because the 4 in the tens column represents 40. (It will always be the case that zero needs to be recorded in the ones column in this part of the procedure, so the zero can be recorded when starting the second line of working.) I then multiply 3 (which represents 3 tens or 30) by 2 (which represents 2 tens or 20), and record 6 (which represents 600, or 30 by 20) in the hundreds column.’*Step 8:* The teacher models this step of the procedure by pointing to numbers as they are used, and recording the results. The teacher says, ‘Now I add the two numbers in my lines of working together using a [formal written algorithm](http://educationstandards.nsw.edu.au/wps/portal/nesa/k-10/learning-areas/mathematics/special-needs-in-mathematics-guide/assessment#operation_errors_wrong) for addition to get the answer to the problem, 672’.*Step 9:* The teacher reiterates the procedure: ‘First I multiplied 32 by 1 and got 32. Then I multiplied 32 by 20 and got 640. Then I added the 32 and 640 together and got the answer 672. Jason swam 672 laps in 21 days.’*Part B* |
|  | Adjustments for Robert |
| The teacher provides [guided practice](http://educationstandards.nsw.edu.au/wps/portal/nesa/k-10/learning-areas/mathematics/support-materials-for-students-with-disability-k-6/implementation/1-procedures#guidedprac) for the procedure, by asking questions about each step of the procedure. The teacher either rephrases or corrects student responses to the questions.*Step 1:* The teacher poses the problem: ‘There are 23 teams in a soccer competition. With 12 players on each team, what is the total number of players in the competition?’*Step 2:* The teacher asks, ‘What operation should I use to solve the problem?’*Step 3:* The teacher asks, ‘What procedure can I use to solve the problem?’*Step 4:* The teacher asks, ‘What should I do before using the procedure, so that I can check my answer?’*Step 5:* The teacher says, ‘Now I am going to record the problem.‘Which number should I write first?’‘Where should I write the other number?’‘Where do I write the multiplication sign?’*The teacher writes the following on the board:**23 x 12* | * the teacher provides Robert with immediate feedback
* the teacher might need to repeat guided practice with Robert, with additional questions
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| *Step 6:* The teacher asks:‘What do we do for the first line of working?’‘Which numbers do I multiply first?’‘What is 3 multiplied by 2?’‘What does the 6 represent?’‘Where do I record the result?’*The teacher records the result.*‘Which numbers do I multiply next?’‘What is 2 multiplied by 2?’‘What does the 4 represent?’‘Where do I record the result?’*The teacher records the result.*‘What is 23 multiplied by 2?’*Step 7:* The teacher asks:‘What do we do for the second line of working?’‘What do I record in the ones column of the second line of working?’‘Why?’‘What numbers do I multiply first?’‘What is 3 multiplied by 1?’‘What does the 3 represent?’‘Where do I record the result?’*The teacher records the result.*‘What numbers do I multiply next?’‘What is 2 multiplied by 1?’‘What does the 2 represent?’‘Where do I record the result?’*The teacher records the result.*‘What is 23 multiplied by 10?’*Step 8:* The teacher asks:‘What do I do next?’‘What procedure do I use to add the results?’*The teacher guides the students in adding 46 and 230 using a formal written algorithm for addition.*‘How many players are there in the soccer competition?’*Step 9:* The teacher reiterates the procedure. | * given that Robert has demonstrated that he can multiply two-digit numbers by one-digit numbers, the teacher could ask Robert questions about *Step 6* of the procedure to increase his likelihood of success
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| *Part C*Students undertake [independent practice](http://educationstandards.nsw.edu.au/wps/portal/nesa/k-10/learning-areas/mathematics/support-materials-for-students-with-disability-k-6/implementation/1-procedures#independentprac), completing worksheets of problems that can be solved using extended multiplication.*Extension:* The teacher models and guides student practice for multiplying two-digit by two-digit numbers (with trading) and three-digit numbers by two-digit numbers, that can be solved using the extended form of multiplication. For example:Multiplying three-digit numbers by two-digit numbers using the extended form of multiplication | * Robert is provided with grid paper to assist him to align his work
* to assist Robert in his acquisition of the procedure, and so that he experiences a high level of success, the teacher provides him with examples that do not include the following:Multiplication table
* Robert is prompted to use his [multiplication grid](#_Robert’s_multiplication_grid) for multiplication facts that he does not recall readily
* Robert multiplies two-digit numbers by two-digit numbers using the extended form of multiplication fluently, before attempting word problems that require him to use the procedure
* the number of problems completed correctly by Robert is recorded on a graph
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## Robert’s multiplication grid

Robert shades, on his multiplication grid, the multiplication facts (green) (facts making up the 1, 2, 3, 4, 5, 10 times tables) that he knows fluently, and highlights each of the multiplication facts (blue) that he can identify using the commutative property of multiplication, eg 3 × 8 = 24, so 8 × 3 = 24.



# Feedback

The teacher monitors students during the learning experiences and assessment opportunities through:

* observation during guided practice and independent practice
* analysis of the students’ work samples.

The teacher monitors the number of problems completed by Robert during independent practice.

The teacher provides specific feedback, such as:

* ‘You have set out your work correctly, making sure that the ones, tens and hundreds in the bottom number are under the ones, tens and hundreds in the top number. The bottom number has no hundreds so nothing is recorded in the hundreds column.’
* ‘In your first line of working, you correctly multiplied 4 by 6 to get 24. You recorded 4 in the ones column of your first line of working and the two tens above the number of tens in the top number of the two numbers you were multiplying. You correctly multiplied 1 by 6 to find 6 but in recording the number of tens in the tens column in your first line of working, you forgot to add the 2 tens. Correct your first line of working by adding the 2 tens to the 6 tens.’
* ‘Well done Robert, you have followed the procedure correctly for two-digit by two-digit multiplication.’
* ‘Robert, when you multiply numbers using the extended form, you need to record the numbers in your working in the correct columns. The second line of working is for multiplying by tens, so a zero needs to be recorded in the ones column. Use your grid paper to re-work the problem, remembering to record the zero in the ones column of the second line of working.’

# Evidence of learning

The teacher uses an analysis of students’ completed worksheets, anecdotal notes of observations, and checklists of indicators to determine whether students have made progress as a result of the learning experiences.

The number of problems accurately completed determines whether individual students are ready to progress to the fluency phase of learning for multiplication using the extended form (long multiplication).

## Criteria for assessment

Students are assessed on their ability to:

* apply appropriate mental or written strategies to solve multiplication problems
* multiply two- and three-digit numbers by two-digit numbers using the extended form of multiplication
* use problem-solving strategies, including those based on selecting and organising key information in a systematic way
* make generalisations about number relationships.

# Evaluating

The teacher makes judgements about the effectiveness of the teaching program based on the evidence of learning. This informs future [programming](http://educationstandards.nsw.edu.au/wps/portal/nesa/k-10/learning-areas/mathematics/special-needs-in-mathematics-guide/programming) and instruction.

The teacher evaluates Robert’s setting out of work following his use of the grid paper provided, the data recorded on his graph, and his on-task behaviour.